Channel Selection and Coordination in Dual-Channel Supply Chains

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Abstract

This paper investigates the influence of channel structures and channel coordination on the supplier, the retailer, and the entire supply chain in the context of two single-channel and two dual-channel supply chains. We extensively study two Pareto zone concepts: channel-adding Pareto zone and contract-implementing Pareto zone. In the channel-adding Pareto zone, both the supplier and the retailer benefit from adding a new channel to the traditional single-channel supply chain. In the contract-implementing Pareto zone, it is mutually beneficial for the supplier and the retailer to utilize the proposed contract coordination policy. The analysis suggests the preference lists of the supplier and the retailer over channel structures with and without coordination are different, and depend on parameters like channel base demand, channel operational costs, and channel substitutability.

Keywords: Channel selection; Channel competition; Channel coordination; Pareto zone

Introduction

Many suppliers face a channel distribution decision of whether to add a new retailing channel in addition to their existing channels. For instance, with the popularity of the Internet, many top suppliers, such as IBM, Cisco, Nike, and Estee Lauder, have started selling directly online. In 2006 the top PC supplier, Dell, opened a retail store in Dallas and another one in New York, in addition to its popular online direct channel. Several electronics suppliers, including Sony, PalmOne, and Samsung, have set up boutique-style outlets in upscale locations.

The addition of a new channel will likely cause channel conflict. As claimed in a letter sent by Home Depot to more than 1000 of its suppliers (Brooker 1999), if those suppliers add direct channels, Home Depot has “the right to be selective in regard to the vendors we select ... a company may be hesitant to do business with its competitors.” While online direct channels can yield more profits for suppliers (Chiang, Chhajed, and Hess 2003; Tsay and Agrawal 2004a), methods to alleviate retailers’ concerns and enhance multichannel supply chain performance have been topics of interest for both academia and industry (Allen 2000). Theoretically, channel coordination can yield more profits to retailers, thus, channel conflict can be reduced (Cachon 2003; Ingene and Parry 2004). However, the extant literature has a theoretical gap in how channel selection influences interactions among suppliers and retailers, which motivates us to ask the following research question: How do different supply chain structures affect a supplier’s channel selection, the retailer’s profit, and overall supply chain efficiency with and without coordination?

Our work is the first to provide comparisons among the following four supply chain structures with and without coordination: a traditional single-supplier-single-retailer retail channel (Scenario R), a supplier-owned direct channel (Scenario D), a dual-channel supply chain with a retail channel and a direct channel (Scenario RD), and a dual-channel supply chain with two retail channels (Scenario RR). A single supplier, as the Stackelberg game leader, is selling the same product in the above four channel scenarios. All these channel scenarios have been studied in the literature; however, none has systematically compared them in terms of channel selection and coordination. In this work, we present two main themes that are associated with whether the supplier and the retailer in Scenario R can benefit from the introduction of a new channel, either direct or retail, with and without contract coordination.

In the first main theme, without coordination, we identify a channel-adding Pareto zone where both the supplier and the retailer can benefit if the supplier introduces a new direct chan-
nel (Scenario RD). Indeed, a similar Pareto zone in Scenario RD is first reported by Chiang, Chhajed, and Hess (2003) and then supported by Arya, Mittendorf, and Sappington (2007). Our paper explores the same feature from a different perspective with asymmetric base demand in dual channels and extends it to the situation under coordination. We further demonstrate that a supplier may encounter a hazard by adding a direct channel when supply chains are coordinated. This counterintuitive result occurs when the retailer is significantly more powerful than the new direct channel (e.g., the direct channel operational cost is relatively high), such that the supplier’s gain in the direct channel cannot compensate for the loss in the retail channel due to more intense competition. However, no Pareto zone has been observed when the supplier adds a new retail channel. Although the incumbent retailer benefits from less double marginalization, the yield is not sufficient to overcome the encroachment of the new retailer.

Based on the above first theme, we further illustrate the channel selection preference of the supplier and the retailer. Under a symmetric setting where the dual channels are equally powerful, the supplier’s preference sequence is given by (from most favorable to least): Scenarios RD, D, RR, and R. This result is attributable to two factors: first, adding a direct channel results in more dominance power for the supplier, and second, the supplier benefits from more intense competition. On the other hand, the incumbent retailer’s preference sequence is: Scenarios R, RR, RD, and D. However, the above preference sequences alter as the channel structure becomes asymmetric.

The second main theme is that negotiation power between the supplier and the retailer embedded in the contract, exemplified by revenue sharing contracts (RSC) in this paper, is changing across Scenarios R, RD, and RR. As indicated by Cachon and Lariviere (2005), there exists a contract-implementing Pareto zone for implementing an RSC in Scenario R, where both the supplier and the retailer can benefit by implementing the contract. Our paper extends the existing literature to quantify and compare the contract-implementing Pareto zones for all three scenarios, R, RD, and RR. The analysis points out that, in terms of the contract-implementing Pareto zone, the retailer prefers RR first, R second, and RD third; while the supplier prefers RD first, R second, and RR third. This result occurs because the supplier is more dominant in RD than in R and RR, but has to transfer more profit to the retailers in RR due to the coexistence of two retailers.

The remainder of this paper is organized as follows. We review the literature in ‘Literature review’ section and introduce the model in ‘The model’ section. We perform the analysis of channel selection and channel-adding Pareto zone in ‘Channel selection without coordination’ section and then discuss channel coordination and contract-implementing Pareto zone in ‘Channel coordination’ section. We conclude in ‘Conclusion’, and all proofs are given in Appendix A.

**Literature review**

This paper focuses on channel selection and coordination in a dual-channel supply chain. Thus, related literature includes multichannel supply chain competition and coordination. The literature on multichannel supply chains involving a direct channel has been dedicated to determining whether a supplier should add a direct channel to its existing retail channel. According to Chiang, Chhajed, and Hess (2003), it is beneficial for a supplier to set up a direct channel to compete with its retailer in a model, assuming that consumers have a common positive preference for the local retailer. Chiang, Chhajed, and Hess (2003) also reports a Pareto zone where both the supplier and the retailer can be better off after the supplier enters the direct channel. The same conclusion is further demonstrated in Arya, Mittendorf, and Sappington (2007). Our paper follows this trend but from a different perspective with asymmetric base demand in two channels and explores this feature in situations with and without coordination.

Indeed, there has been a large volume of literature focused on channel competition. In a duopoly common retailer channel model, Choi (1996) demonstrates the differences among three game settings, including two Stackelberg games and a vertical Nash game. In a seminal work on a dual exclusive channel, McGuire and Staelin (1983) provide an explanation on why a supplier would want to use an intermediary retailer in the context of two supply chains with one supplier in each chain. Through the theory of channel control, Bucklin (1973) suggests the degree of coordination among players is a measure of the competitive position of that supply chain from the perspectives of payoff, middleman tolerance, and others. El-Ansary (1974) relaxes some assumptions of Bucklin (1973) and points out that the balanced point of channel power is the interactive result of the channel members. Etgar (1978) empirically suggests a channel control mix aiming for a proper and efficient design of channel control tools for leaders. Other related multichannel literature also studies channel conflict and coordination (Brown 1981; Brown, Lusch, and Muehling 1983; Lusch 1976), inventory control (Boyaci 2005; Chiang and Monahan 2005; Dong, Dresner, and Shankar 2007), service competition (Dumrongsiri et al. 2008; Tsay and Agrawal 2004a), outlet malls (Coughlan and Soberman 2005), channel distribution (Bernstein, Song and Zheng 2008; Kumar and Ruan 2006; Zettelmeier 2000), direct channel entry deterring (Liu, Gupta, and Zhang 2006), and others (Fay 2008; Lueg et al. 2006; Tsay 2002; van Birgelen, de Jong and de Ruyter 2006; Wallace, Giese, and Johnson 2004). In a recent discussion, Brown et al. (2005) provide insights into how supply chain management contributes to the “Big Middle.” Observing that sale leakage occurs to retailers who sell through multiple channels, Yuan and Krishna (2008) suggest that different revenue models have their own advantages; for example, a fixed monthly fee can be more profitable than an all-revenue-share fee for a mall or e-mall, like Yahoo!Shopping. A comprehensive review of multichannel models can be found in Cattani, Gilland, and Swaminathan (2004) and Tsay and Agrawal (2004b). However, the above literature has not explic-
situations of coordination. In this paper, we utilize the revenue
nel on the supplier, the retailer, and the entire supply chain, in
supply chain structures, especially the impact of a direct chan-
Moreover, few papers have focused on the efficacy of different
cooperation of a dual-channel supply chain including a direct chan-
the above literature has not explicitly addressed full coordina-
and Ingene and Parry (2004) for an insightful dis-
interested readers are
Tsay, Nahmias, and Agrawal (1999) for surveys of contracts
also referred to Ingene and Parry (1995a) demonstrate that a two-part tariff wholesale pricing policy can fully coordinate the channels. Ingene and Parry (1995b) also point out that the manufacturer, however, will prefer the second-best two-part tariff to a menu of two-part tariffs maximizing the channel profits. Raju and Zhang (2005) show that a manufacturer would choose one contract (i.e., quantity discounts or two-part tariffs) over the other in the presence of a dominant retailer. Cachon and Lariviere (2005) apply a revenue sharing contract to coordinate the supply chain with a supplier and a retailer or multiple symmetric retailers competing in quantities. In their model, the supplier and the retailer agree on the revenue sharing percentage and the wholesale price before the retailer determines the optimal order quantity and retail price. They also compare the revenue sharing contract to others and demonstrate that the revenue sharing contract can coordinate a broad array of supply chains. Indeed, many other contract forms have been widely discussed in recent years. One can refer to Cachon (2003) and Tsay, Nahmias, and Agrawal (1999) for surveys of contracts for a wide range of supply chain models. Interested readers are also referred to Ingene and Parry (2004) for an insightful discussion on channel distribution and coordination. Nevertheless, the above literature has not explicitly addressed full coordination of a dual-channel supply chain including a direct channel. Moreover, few papers have focused on the efficacy of different supply chain structures, especially the impact of a direct channel on the supplier, the retailer, and the entire supply chain, in situations of coordination. In this paper, we utilize the revenue
sharing contract to demonstrate that negotiation power between the supplier and the retailer varies over different supply chain structures.

The model

We study and compare four different supply chain structures/scenarios as illustrated in Fig. 1. Scenario R represents a traditional supply chain structure in which a supplier sells products exclusively through a retailer. In Scenario D, a supplier (e.g., Dell before 2006) sells products exclusively through a direct channel to consumers. Scenarios R and D are studied as two benchmark cases.2 Scenario RD is a hybrid model of Scenarios R and D, which are exemplified by many click-and-mortar suppliers, such as PC suppliers like Apple and HP or service providers like major airline companies. In Scenario RR, a supplier sells the product through two retailers. In practice, many suppliers still utilize either Scenario R or RR which can be exemplified by the thousands of suppliers of Wal-Mart and Home Depot. It is worth noting that the direct and retail channels could be either online or offline. Without loss of generality, we assume that the first retailer in Scenarios R, RD, and RR is the same retailer, referred to as Retailer r or the (incumbent) retailer; the second retailer in Scenario RR is referred to as Retailer r2 or the new retailer. The supplier is referred to as Supplier s.

Because Scenarios RR and RD share the same features at the retail level, the channel demand functions in these two scenarios are the same. To avoid confusion in notation, we discuss in ‘Dual-retailer: Scenario RR’ section.

We assume that there is only one kind of product for sale. We use $D_r$ to represent demand in the direct channel and $D_t$ in the retail channel. The direct channel price is denoted by $P_s$, and the retail (channel) price $P_r$. To obtain the demand functions in different channel structures, we adopt the elegant framework established by Ingene and Parry (2004) (Chapter 11) and Ingene and Parry (2007) and employ a similar utility function for a

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2 Theoretically, Scenario D can be considered as an integrated or coordinated Scenario R; however, in our model setting, Scenarios R and D differ in both channel base demand and channel operational cost as shown in the sequel.
representative consumer as follows\(^3\):

\[
U = \sum_{i=s,r} \left( \alpha_i D_i - \frac{b_i D_i^2}{2} \right) - \theta D_s D_r - \sum_{i=s,r} P_i D_i,
\]

where \(\alpha_i\) denotes the base demand (intercept) of channel \(i, i = s, r\); parameter \(b\) denotes the rate of change of marginal utility and is normalized to one in the sequel for brevity; and \(\theta\) \((0 \leq \theta < 1)\) denotes channel substitutability. The channels are demand interdependent (unless \(\theta = 0\)), although \(\alpha_s\) and \(\alpha_r\) appeal to different market segments. The aggregate demand decreases in \(\theta\).

In Scenario RD, the supplier adds a direct channel and competes with the retailer. Maximization of Eq. (1) yields the demand for each channel as follows:

\[
D_s = \frac{\alpha_s - \theta \alpha_s - P_s + \theta P_r}{1 - \theta^2},
\]

\[
D_r = \frac{\alpha_r - \theta \alpha_r - P_r + \theta P_s}{1 - \theta^2}.
\]

In Scenario R, the supplier sells the product exclusively through the retailer, thus, \(D_s = 0\). Maximization of Eq. (1) yields

\[
D_s = 0, \quad D_r = \alpha_r - P_r.
\]

Similarly in Scenario D, the supplier owns and sells the product exclusively through the direct channel \((D_s = 0)\), thus,

\[
D_s = \alpha_s - P_s, \quad D_r = 0.
\]

Parameter \(\Pi_s\) denotes the supplier’s profit, \(\Pi_r\) the retailer’s profit, and \(\Pi\) is the total profit for the entire supply chain. The operational cost in the direct channel is \(c_s\) and the retail channel \(c_r\), regardless of whom is operating the channels. The wholesale price is \(w\). For brevity, we normalize the production cost to zero. To facilitate our discussion, we define

\[
\Omega = \frac{\alpha_r - c_r}{\alpha_s - c_s}
\]

as the relative channel power of channel \(r\) over channel \(s\) in the dual-channel cases. If \(\Omega > 1\), channel \(r\) is superior to channel \(s\), and vice versa. In scenarios R, D, and RD, the retailer’s and supplier’s profit functions can be written as

\[
\Pi_r = D_r (P_r - w - c_r),
\]

\[
\Pi_s = D_s w + D_s (P_s - c_s).
\]

We impose some constraints. Channel prices must exceed marginal costs such that \(P_r \geq w \geq c_r \geq 0\) and \(P_s \geq c_s \geq 0\). In order to have meaningful comparison among channel structures, all channel demands must be nonnegative, such that \(\Omega \geq \theta\) \(^4\); and for the single-channel cases, we have \(\alpha_i \geq P_i \geq c_i, i = r, s\).

We adopt a supplier-Stackelberg leader game in all scenarios. In Scenario R, the supplier determines the wholesale price first and the retailer determines the retail price afterwards. In Scenario D, the supplier determines the direct channel price to optimize supply chain profit. In Scenario RD, the supplier determines the wholesale price in the first stage; in the second stage, the supplier determines the direct channel price and the retailer determines the retail price simultaneously in a Nash game. We solve the game by backward induction. Similar game configurations have been used in the literature (see McGuire and Staelin, 1983, 1986).

In order to coordinate the supply chains, especially in Scenarios R, RD, and RR, we primarily investigate a revenue sharing contract (RSC), in which the retailer shares \(\rho\) percentage of its revenue with the supplier. For example, in Scenarios R and RD with RSC, Eq. (5) becomes

\[
\Pi_r = D_r ((1 - \rho) P_r - w - c_r),
\]

\[
\Pi_s = D_s (\rho P_s + w) + D_s (P_s - c_s).
\]

The wholesale price is described as \(w = \delta - \rho c_r\), where \(\delta\) is a contract adjustment factor.\(^5\) The value of \(\rho\) generally reflects the negotiation power between the supplier and the retailer and is usually determined in a negotiation process. Although we will show in ‘Channel coordination’ section how different supply chain structures affect the contract-implementing Pareto zone (a mutually beneficial range for both the supplier and the retailer by applying the contract) in terms of \(\rho\), the negotiation process of \(\rho\) is not the focus of this paper. The RSC has been popular in practice and theory. One well-known example is the coordination between Blockbuster Inc., a video retailer, and its suppliers, in which Blockbuster shares a percentage (estimated in the range of 30–45%) of its revenue with the suppliers in return for a sharp drop in the wholesale price from $65 to $8 per tape (Cachon and Lariviere, 2005). An RSC policy has also been used in an exclusive deal between AT&T Wireless and Apple, the provider of the wireless phone iPhone, in which AT&T shares a portion of its monthly rate with Apple for every purchased iPhone. In theory, Cachon and Lariviere (2005) show that an RSC can fully coordinate the supply chain with a supplier and a retailer or multiple symmetric retailers competing in quantities. Different from Cachon and Lariviere (2005), we demonstrate that the variants of the RSC can coordinate both a supply chain with two retailers competing in prices and a dual-channel supply chain including a direct channel. It is worth noting that our focus is not only on how the RSC coordinates different supply chains, but also on how different supply chain structures affect the negotiation power between the supplier and the retailer under coordination.

**Channel selection without coordination**

This section focuses on the selection of a specific channel structure without coordination. Without coordination, the players optimize their own profits sequentially rather than maximize

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\(^3\) The author is very grateful to an anonymous reviewer for suggesting this utility function. Interested readers may refer to Ingene and Parry (2004) (Chapter 11) for more properties regarding this utility function.

\(^4\) The upper bounds of \(\Omega\) are different in Scenarios RD and RR and are provided in Appendix A.

\(^5\) The format of \(w\) is for the purpose of parsimony in computation. It is equivalent to consider \(w\) directly without the transformation.
the entire supply chain profit globally. Due to different supply chain structures, the supply chain players behave differently in Scenarios R, D, RD, and RR. The optimal solutions for Scenarios R, D, and RD without coordination are summarized in Table A.1 in Appendix A.

We first consider Scenarios R and D. Under the symmetric setting, which is defined as $\alpha_r = \alpha_s$ and $c_r = c_s$ (i.e., $\Omega = 1$) throughout this paper, Scenario D has an advantage for the supplier over Scenario R. This result is intuitive because Scenario D eliminates the intermediary; while in Scenario R the retailer raises the retail price, which causes a double marginalization problem. Removing the above symmetric setting, we obtain the boundary condition (i.e., $\Omega < \sqrt{2}$) where Scenario D outperforms Scenarios R for the supplier. This result indicates that Scenario D still has an advantage over Scenario R even if the retail channel is more powerful (as long as $\Omega < \sqrt{2}$). However, not every supplier can successfully warrant sufficiently high relative channel power (e.g., low operational cost and/or a high base demand), when operating a direct channel. For example, Levi Strauss and Company stopped selling online in 1999 due to high operational costs and low demand.6

The impact of a direct channel: Scenario RD

Comparison: RD versus R

In Scenario RD, the retailer has to compete with the direct channel owned by the supplier. A question arises whether the supplier and the retailer would be better off in RD than in R. Comparing the corresponding profits in Scenarios R and RD, we obtain the following result.

**Theorem 1.** For the supplier, Scenario RD outperforms Scenario R; for the retailer, Scenario RD outperforms R if

$$\Omega > \hat{\Omega}_1 \equiv \frac{\theta(8 + 2\theta^2)}{8 + 4\theta^2 - (8 + \theta^2)\sqrt{1 - \theta^2}} (> 1),$$

where $\hat{\Omega}_1$ is decreasing in $\theta$.

Theorem 1 demonstrates that the supplier benefits from adding a direct channel. This is intuitive because the existence of the direct channel not only forces the retailer to reduce the retail price, which results in higher demand, but also generates more profits for the supplier by owning the direct channel. However, it is somewhat counterintuitive that the retailer can be better off as well, after the supplier adds the direct channel. This observation concurs with Chiang, Chhajed, and Hess (2003) and Arya, Mittendorf, and Sappington (2007) but from a different perspective with asymmetric base demand in dual channels. As shown in Theorem 1, the retailer can perform better in Scenario RD than in Scenario R when the retail channel has a sufficient advantage over the direct channel ($\Omega > \hat{\Omega}_1$). Such an edge increases when channel substitutability increases, because the supplier charges a lower wholesale price in Scenario RD than in Scenario R, and double marginalization is lessened due to direct channel competition.

The phenomenon where both the supplier and the retailer are better off in Scenario RD than in Scenario R is referred to as Pareto efficiency of adding a new channel in dual-channel competition. The range of the Pareto efficiency is referred to as a channel-adding Pareto zone. Since the retailer is not always better off in Scenario RD than in Scenario R, a concern naturally arises: should the supplier transfer partial payment to the retailer to obtain the Pareto efficiency? In practice, the answer might depend on the negotiation power of the retailer relative to the supplier. For instance, in the previously mentioned Home Depot case, if the supplier very much relies on Home Depot, it would be beneficial for the supplier to transfer partial payment to the retailer; otherwise, the supplier may suffer significant losses if the retailer switches to other suppliers. On the other hand, if the supplier is very powerful, such as IBM computer servers, the supplier may be reluctant to transfer payment to the retailer, since the retailer has few alternative suppliers.

Comparison: RD versus D

While Scenario RD outperforms R for the supplier, an additional question is whether the supplier would like to introduce a retail channel given Scenario D. Comparing the corresponding profits in Scenarios D and RD leads to the following result.

**Theorem 2.** For the supplier, Scenario RD outperforms D.

Theorem 2 indicates that the supplier is better off selling through a new retailer when owning the incumbent direct channel. As channel substitutability decreases, the two channels become more monopolistic. Consequently, the supplier gains additional benefit from wholesaling to a new retailer while maintaining a significant profit from the incumbent direct channel, although the supplier might lose some edge in the direct channel due to competition from the retail channel. The result of Theorem 2 is supported by the practice of Dell. Before 2006, Dell had only the (online) direct channel (Scenario D); however, in 2006, Dell started to sell products through retailers such as Walmart (Scenario RD).7

Dual-retailer: Scenario RR

In Scenario RR, the supplier sells a product through two retailers. The direct channel in Scenario RD is replaced with a new retailer in Scenario RR. We call this new retailer Retailer $r_2$ relative to the incumbent retailer (Retailer $r$) in Scenarios R and RD. In order to compare Scenario RR with Scenario RD, we assume the new retail channel has the same market power as the direct channel in Scenario RD. In addition, we slightly abuse

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6 The high operational costs and low demand for Levis Strauss and Company were attributed to immature Internet technology and the difficulty of attracting online consumers back in 1990s. Thanks to the rapid development of Internet technology and growing popularity of online shopping, Levis Strauss and Company recently reopened its online store.

7 Although our analysis provides theoretical support for Dell’s expansion to local retailers, another important reason might be that Dell’s competitors entered the online market without much difficulty, which encroached on much of Dell’s original online market share. A multi-supplier model will be a priority in future research to more comprehensively explain the above strategic move of Dell.
the notation by using the same notation of the direct channel for Retailer \( r2 \) (e.g., \( P_r \) denotes the retail price of Retailer \( r2 \)). The wholesale prices to Retailers \( r \) and \( r2 \) are denoted as \( w_r \) and \( w_{r2} \), respectively. Similarly, channel operational costs continue to be \( c_r \) and \( c_{r2} \), but for Retailer \( r \) and \( r2 \), respectively. Due to the same competition feature at the retail level, the demand functions for the two retailers in Scenario RR are the same as in Scenario RD, which are written as follows:

\[
D_{r-RR} = \frac{\alpha_r - \theta \alpha_s - P_r + \theta P_s}{1 - \theta^2},
\]

\[
D_{r2-RR} = \frac{\alpha_s - \theta \alpha_r - P_s + \theta P_r}{1 - \theta^2}.
\]

The profit functions for the two retailers and the supplier, respectively, become

\[
\Pi_r = D_{r-RR}(P_r - w_r - c_r),
\]

\[
\Pi_{r2} = D_{2-RR}(P_s - w_s - c_{r2}),
\]

\[
\Pi_s = D_{r-RR}w_r + D_{2-RR}w_{r2}.
\]

Comparison: RR versus R

When there is no coordination between the supplier and the retailers, the supplier, as the Stackelberg leader, determines wholesale prices in the first stage, and then the retailers simultaneously determine their respective retail prices in a Nash game in the second stage. We first compare Scenario RR with Scenario R and obtain the following result.

**Lemma 1.** For the supplier, Scenario RR outperforms Scenario R; while for the retailer, Scenario R outperforms Scenario RR.

We attribute Lemma 1 to the existing literature (e.g., Inge and Parry (2004)). It is intuitive that a supplier can benefit from adding a new retail channel, because more intense competition exists at the retail level in Scenario RR than in Scenario R, thus, Scenario RR will capture more consumers and enable the supplier to charge a relatively higher wholesale price. On the other hand, because a new retailer is less monopolistic than a new direct channel, the incumbent retailer is relatively more powerful in Scenario RR than in Scenario RD. It seems that a channel-adding Pareto zone would appear when the supplier adds a new retailer to Scenario R (like in the case of RD); however, this seemingly expected result has never been realized in its entire feasible domain. This is because when the incumbent retailer is sufficiently powerful, the new retail channel is less powerful than the direct channel in lessening the double marginalization; consequently the retail price is not undercut enough that the incumbent retailer’s profit erodes more in Scenario RR than in RD. Meanwhile, compared with a direct channel, the new retail channel is more vulnerable and will be pushed to the brink in order for the incumbent retailer to reach the Pareto zone. Notwithstanding, even if the new retail channel is priced out of the market (demand equals zero), the incumbent retailer cannot yield a sufficiently high profit, unless channel substitutability is zero where both channels become monopolists.

Comparison: RR versus D

Comparing Scenario RR to Scenario D yields the result as follows.

**Theorem 3.** For the supplier, Scenario RR outperforms D if

\[
\Omega \geq \Omega_2 \equiv \frac{\theta + (1 - \theta^2) \sqrt{4 - \theta^2}}{2 - \theta^2} (> 1).
\]

Otherwise, Scenario D outperforms RR.

Theorem 3 suggests that Scenario RR can outperform Scenario D when the direct channel is relatively less powerful. This result is intuitive because the supplier can benefit from a larger demand from dual retail channels, although the marginal profit is lower in RR than in D. On the other hand, Scenario D can outperform Scenario RR when the direct channel is powerful enough that the benefit of a higher marginal profit in a single direct channel outpaces the gain from a larger demand from two channels. The latter result holds even if the channel setting is symmetric (\( \Omega = 1 \)) and, hence, demonstrates the benefit of integrating a channel. This, again, might partially explain why Dell could become so successful even without relying on retailers before 2006.9

Choice of channel structures

In the above discussion, we have compared Scenarios RD and RR to Scenarios R and D. We are now in a position to compare all these channel structures for the supplier and the retailer. We start from the symmetric setting and then provide more discussion that follows. We first provide the following observation.

**Theorem 4.** Given the symmetric setting (\( \Omega = 1 \)), channel selection preferences for the supplier and the (incumbent) retailer from most to least favorable are given as follows.

The supplier : \( RD > D > RR > R \);

The retailer : \( R > RR > RD > D \).

For the supplier, the observation of \( RR > R \) is directly from Lemma 1. The result of \( D > RR \) can be obtained from Theorem 3 since \( \Omega = 1 < \hat{\Omega}_2 \). For the retailer, that \( D \) is the least preferred by the retailer is straightforward, since the retailer earns zero in Scenario D. The result of \( R > RR \) for the retailer is directly from Lemma 1. The outcome of \( RD > RR \) for the supplier is intuitive because the supplier has more market power in Scenario RD than in Scenario RR, and the reverse (\( RR > RD \)) is true for the incumbent retailer.

We now exhibit the case where \( \Omega \neq 1 \) through the following example.

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8 The assumption of different wholesale prices is in line with Cachon and Lariviere (2005); Bernstein and Federgruen (2005) and Dukes, Gal-Or, and Srinivasan (2006), and is needed in order to fully coordinated the dual-retailer channel.

9 Dell became the No. 1 PC maker in 2001, but lost the top position to HP in 2009 partially because of the economic crisis starting in 2008.
Numerical example

We first demonstrate the supplier’s channel selection. To demonstrate the impact of $\Omega$ ($\Omega = ((\alpha_r - c_r) / (\alpha_s - c_s))$), we choose $c_s$ as the independent variable while fixing the values of other parameters. As $c_s$ increases, $\Omega$ increases.

As shown in Fig. 2, when relative channel power is small (i.e., $c_s < 2.65$ or $\Omega < \hat{\Omega}_2$), the supplier’s preference sequence is $RD > D > RR > R$, which is consistent with Theorem 4 since $\Omega = 1$ is a special case of this situation. As $c_s$ grows, relative channel power becomes more asymmetric ($\Omega > \hat{\Omega}_2$), especially as the incumbent retailer (channel $r$) becomes more powerful. Thus, Scenario $RR$ starts to dominate Scenario $D$ and the new preference list is given as $RD > RR > R > D$. If $c_s$ continues to increase such that $\Omega > \sqrt{2}$, then Scenario $R$ outperforms Scenario $D$, thus, the supplier’s preference is $RD > RR > R > D$. As $c_s$ continues to grow such that $\Omega > \tilde{\Omega}_2$ as shown in Fig. 2, we observe an interesting result in that the supplier prefers Scenario $RR$ to Scenario $RD$. This result occurs because, in Scenario $RD$, the supplier charges a high direct channel price due to sufficiently high operational cost; consequently, direct channel demand declines to a level converging to zero below corresponding demand (of channel $s$) in Scenario $RR$. Although the feasible domain of $\Omega > \tilde{\Omega}_2$ is relatively small, this observation suggests the supplier could be better off by delegating sales to a retailer if direct channel power is very weak.

For the retailer, if relative channel power is small (i.e., $\Omega < \hat{\Omega}_2$), the channel preference sequence is $R > RR > RD$, which is consistent with Theorem 4 since $\Omega = 1$ is a special case of this situation (Fig. 3). As $c_s$ grows, Scenario $RD$ outperforms Scenario $RR$ for the retailer during $\hat{\Omega}_2 \leq \Omega \leq \hat{\Omega}_1$. As $\Omega \geq \hat{\Omega}_1$, we observe the channel-adding Pareto zone as described in Theorem 1, because a more powerful retail channel enables the retailer to attract enough customers and avoid a severe pricing war with the direct channel as direct channel operational cost builds.

We finally compare all scenarios in terms of overall supply chain efficiency. Through a representative graph, Fig. 4, we show that Scenario $RD$ outperforms all other scenarios. This is because the entire supply chain benefits from a larger market, relative to Scenarios $R$ and $D$, and an integrated channel, relative to Scenario $RR$. Consistent with Theorem 3, Scenario $D$ outperforms Scenario $RR$ when $\Omega$ is relatively small, but vice versa as $\Omega$ grows. The result that Scenario $R$ outperforms Scenario $D$ when $c_s$ becomes big enough ($c_s > 3.9$) is intuitive and consistent with our comparison between Scenarios $R$ and $D$. We also find that an integrated channel yields more overall supply chain profit than the same non-integrated one, which leads us to investigate channel coordination in the following section.

Channel coordination

In this section, we study channel coordination in Scenarios R, RD, and RR. We start at Scenario R and see the revenue sharing policy can fully coordinate Scenario R. The optimal retail price is the same as the direct channel price in Scenario D when the retail and direct channels are symmetric, which suggests that

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10 It is equivalently effective to demonstrate our results by selecting another parameter, such as $c_r$, $\alpha_r$, or $\alpha_s$.

11 The value of $\hat{\Omega}_2$ can be obtained by equating the supplier’s profits in Scenarios RD and RR.

12 The value of $\hat{\Omega}_2$ can be obtained by equating the retailer’s profits in Scenarios RD and RR.

13 When $\Omega > \hat{\Omega}_3$, Retailer $r$’s gain from Scenario RD over RR outweighs the supplier’s loss from Scenario RD over RR.
coordination eliminates double marginalization in this single-channel scenario. By comparing the supplier’s and retailer’s profits when coordinated versus uncoordinated, we obtain the following result.

**Lemma 2.** In Scenario R, it is mutually beneficial for both the supplier and the retailer to apply coordination mechanism as long as

\[
\frac{1}{2} < \rho < \frac{3}{4}.
\]

(14)

The domain defined by Eq. (14) is referred to as a contract-implementing Pareto zone for Scenario R, where both the supplier and the retailer yield more profits under coordination. To ensure both the supplier and the retailer will participate in the coordination, the revenue sharing rate should be in a reasonable range (as required by Eq. (14)); otherwise, the disadvantaged party will deviate from the coordination. More specifically, if the supplier’s negotiation power is too high (\(\rho > 3/4\)), the retailer will not cooperate; on the other hand, if the retailer’s negotiation power is too high (\(\rho < 1/2\)), the supplier will deviate.

**Direct channel coordination: Scenario RD**

We now show that a coordination policy, which is a combination of a revenue sharing contract and a pricing scheme \(P_r = P_s + \varepsilon\), can fully coordinate Scenario RD. To coordinate a supply chain that includes more than one channel, the coordination policy goes beyond eliminating double marginalization as in a single-channel supply chain, to accounting for inter-distribution competition between channels. Theoretically, a coordinated Scenario RD performs as well as an integrated Scenario RD, where the supplier owns both the direct and retail channels and simultaneously determines channel prices. By applying the above coordination policy to Scenario RD, we obtain the following result.

**Theorem 5.**

1. The coordination policy of \(P_r = P_s + \varepsilon_{RD}\) and the revenue sharing contract with \(w = \delta_{RD} - \rho \delta_i\), fully coordinates Scenario RD.

2. It is mutually beneficial for both the supplier and the retailer to utilize the coordination policy as long as

\[
\rho_{RD} < \rho < \bar{\rho}_{RD},
\]

where

\[
\rho_{RD} = \frac{4 + 4\theta + \theta^3}{8 + \theta^2}
\]

and

\[
\bar{\rho}_{RD} = \frac{48 + 16\theta + 16\theta^3 - 3\theta^4 + 4\theta^5}{(8 + \theta^2)^2}.
\]

**Theorem 5** confirms that the proposed coordination policy can fully coordinate the dual-channel supply chain in Scenario RD. In addition to the effect of the classic revenue sharing mechanism, the components \(\varepsilon\) and \(\delta\) help balance the operational cost difference in the two channels and ultimately coordinate the two competing channels. **Theorem 5** further indicates that the contract-implementing Pareto zone’s lower and upper bounds depend on the channel substitutability level. When the two channels become purely monopolistic (\(\theta = 0\)), the contract-implementing Pareto zone of Scenario RD converges to that of Scenario R. As the channels become more substitutable, both lower and upper bounds increase and converge to 1. Thus, the contract-implementing Pareto zone shifts up relative to that of Scenario R. Therefore, compared with **Lemma 2**, **Theorem 5** suggests that the addition of a direct channel to Scenario R would strengthen the negotiation power for the supplier.

**Two-retailer coordination: Scenario RR and the comparison**

Since there are two retailers in Scenario RR, we utilize a revenue sharing contract for each channel, where \(w_i = \delta_i - \rho \delta_i\), to coordinate the entire supply chain. To ensure tractability, we assume the same revenue sharing rate \(\rho\) for both retailers. We then solve \([P^r, P^s]\) for the retailers in a Nash game, and find \(\delta_i\) and \(\delta_j\) simultaneously to maximize the overall profit of the entire supply chain.

**Theorem 6.**

1. The revenue sharing contracts with \(w_i = \delta_i - \rho \delta_i\), \(i = r, s\), fully coordinates the entire supply chain of Scenario RR.

2. The supplier and the two retailers are better off by complying with the above coordination policy as long as

\[
\rho_{RR} < \rho < \bar{\rho}_{RR},
\]

where

\[
\rho_{RR} = \frac{(2 + 3\theta^2 - \theta^4)(1 + \Omega^2) - 2\theta(5 - \theta^2)\Omega}{(4 + 3\theta^2 - \theta^4)(1 + \Omega^2) - 4\theta(4 - \theta^2)\Omega},
\]

\[
\bar{\rho}_{RR} = 1 - \frac{((2 - \theta^2)\min\{\Omega, 1/\Omega\} - \theta)^2}{(4 - \theta^2)^2(\min\{\Omega, 1/\Omega\} - \theta)^2}.
\]

**Theorem 6** confirms that Scenario RR can be fully coordinated. The coordinated Scenario RR has the same overall supply chain profit as coordinated Scenario RD, and outperforms Scenarios D and R. Similar to Scenario RD, we demonstrate that there exists a contract-implementing Pareto zone such that it is beneficial for all players to utilize the coordination policy. The upper bound \(\bar{\rho}_{RR}\) is derived from both retailers’ upper bounds to warrant a mutually beneficial contract for both retailers, and reaches its maximum at \(\Omega = 1\). On the other hand, for the supplier, the lower bound \(\rho_{RR}\) always reaches its minimum at \(\Omega = 1\) and increases as \(\Omega\) deviates from the symmetric setting. This observation suggests that both the supplier and the retailers can more easily benefit from a coordination contract when the channel structure is more symmetric. The logic behind that finding
is that the supplier can benefit from more intense competition between the retailers when they are more equally powerful; meanwhile, a more symmetric channel structure enables the less powerful retailer to more easily benefit from the contract.14

Comparing the above contract-implementing Pareto zone with those in Scenarios RD and R, we now summarize how the channel structure affects the players’ negotiation power in contracting.

**Theorem 7.** The contract-implementing Pareto zones in Scenarios R, RD, and RR can be characterized as

\[ \rho_{RR} \leq \rho_R \leq \rho_{RD} \quad \text{and} \quad \hat{\rho}_{RR} \leq \hat{\rho}_R \leq \hat{\rho}_{RD}. \]

As discussed previously, the lower bound of the contract-implementing Pareto zone represents the lowest revenue sharing rate at which the retailer would like to share its revenue with the supplier; while the upper bound represents the highest rate at which the retailer would like to share its revenue with the supplier. Since \( \rho_{RR} \leq \rho_R \leq \rho_{RD} \) and \( \hat{\rho}_{RR} \leq \hat{\rho}_R \leq \hat{\rho}_{RD} \), the contract-implementing Pareto zone moves up in the sequence of RR, R, RD. This movement implies that the supplier’s negotiation power over the revenue sharing rate increases; while the retailer’s negotiation power decreases in the sequence of RR, R, RD. Theorem 7 shows that supply chain structure significantly affects the supplier’s and the retailer’s negotiation power in terms of revenue sharing contracts.

**Impact of coordination on channel-adding Pareto zone**

As shown previously, coordination can bring about more profits to both the supplier and the retailer(s) as long as the revenue sharing rate is within the contract-implementing Pareto zone. Both Theorem 1 and Lemma 1 demonstrate that the supplier will be better off by adding a new channel, either a new retailer or a direct channel, to Scenario R. Naturally, we might expect that, under coordination (all scenarios are coordinated under the aforementioned schemes), the supplier would continue to see additional profit from adding a new channel. This is true when the supplier adds a new (coordinated) retailer to Scenario R. That is to say, Lemma 1 holds under coordination.

When the supplier introduces a new direct channel to coordinated Scenario R, we continue to observe the channel coordination zone as suggested in Theorem 1. However, we find a new phenomenon, referred to as supplier’s direct channel addition hazard under coordination, where the supplier can be worse off by adding a direct channel.

**Theorem 8.** Given that both Scenarios R and RD are coordinated with the same revenue sharing rate (\( \rho \)), Scenario RD outperforms Scenario R for the retailer as long as

\[ \Omega > \hat{\Omega}_3 \equiv \frac{\theta}{1 - (1 - \theta)\sqrt{1 + \theta}}. \]

However, Scenario R outperforms Scenario RD for the supplier as long as

\[ \Omega > \hat{\Omega}_4 \equiv \frac{\theta + \theta^2(1 - \rho) - 1}{\theta(\theta - \rho) - (1 - \theta)\sqrt{\theta(1 + \theta)(1 - \rho)(1 - \theta\rho)}}. \]

The channel-adding Pareto zone is given by \( \hat{\Omega}_3 \leq \Omega \leq \hat{\Omega}_4 \). Both \( \hat{\Omega}_3 \) and \( \hat{\Omega}_4 \) are convex decreasing in \( \theta \).

Theorem 8 suggests that the retailer can be better off when it has an advantage in terms of relative channel power (a large \( \Omega \)) over the direct channel, which is consistent with Theorem 1, thus, a similar explanation applies. The advantage is enhanced by coordination, since \( \hat{\Omega}_3 < \Omega_1 \). This outcome suggests that coordination better protects the retailer from the supplier adding a new direct channel. As a result, overall supply chain efficiency improves, and a channel-adding Pareto zone exists for both the supplier and the retailer.15 However, the supplier encounters a direct channel addition hazard under coordination. This phenomenon occurs when the retailer has significantly more channel power than the direct channel (\( \Omega > \hat{\Omega}_4 \)), either because the supplier has much higher channel operational costs or a very small base demand. In this situation, the supplier earns only a small portion of its overall profit from the direct channel, while it has to commit to providing a relatively low wholesale price as required by the contract because of more intense competition. This channel addition hazard does not occur when the supplier adds a new retailer (from coordinated Scenario R to coordinated Scenario RR), because the new retailer provides a cushion as to lessen channel competition. We use the following example to provide a more vivid explanation of Theorem 8.

**Numerical example**

We adopt the same market configuration as in Fig. 3 (without coordination), where \( \alpha_r = \alpha_s = 10 \), \( c_r = 2 \), and \( \theta = 0.3 \), but employ the aforementioned coordination scheme where \( \rho = 0.75 \). Comparing Fig. 5 with Fig. 3, we can infer that it is more likely for the retailer to benefit from adding a new

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14 In other words, the contract-implementing Pareto zone attains its broadest range when \( \Omega = 1 \).

15 Note that \( \hat{\Omega}_3 \) is independent of the revenue sharing rate, which suggests that the contract-implementing Pareto zone persists as long as the supplier benefits from adding a direct channel in the same scope.
direct channel under coordination, versus without, as \( c_j \) grows (i.e., \( \Omega_3 < \Omega_1 \)). However, the supplier would encounter the direct channel addition hazard when the new direct channel becomes sufficiently weaker than the incumbent retail channel (i.e., \( \Omega > \Omega_4 \)), and correspondingly the Pareto zone fades away. Because both \( \Omega_3 \) and \( \Omega_4 \) decrease in \( \theta \) and converge to 1, the channel-adding Pareto zone narrows while the direct channel addition hazard widens as the two channels becomes more substitutable, and vice versa.

**Conclusion**

This paper investigates the impact of channel structure on the supplier, the retailer, and the entire supply chain with and without coordination. Different from previous studies on multichannel supply chains, our paper is the first to explicitly compare the performance of the supplier, the retailer, and the entire supply chain in four different supply chain structures. These structures include a traditional retail channel (Scenario R), a direct channel (Scenario D), a dual-channel supply chain with direct and retail channels (Scenario RD), and a dual-channel supply chain with two retailers (Scenario RR), with and without coordination. Through a revenue sharing contract, we quantify the specific contract formats for different supply chains, and demonstrate the impact of the different supply chain structures on the negotiation power between the supplier and the retailer under coordination.

More specifically, this paper presents two main themes in terms of Pareto zone. The first theme shows that, without coordination, both the supplier and the retailer can mutually benefit when the supplier introduces a new direct channel into the incumbent retail channel (Scenario R), which is referred to as channel-adding Pareto zone. For the supplier, we demonstrate that more channels do not necessarily outperform fewer channels. For example, under some conditions, a single direct channel (Scenario D) can outperform a dual-retailer channel (Scenario RD). We also find the supplier would prefer Scenario RR over Scenario RD, if the new direct channel is sufficiently weaker than the other channel. The second theme is the contract-implementing Pareto zone where both the supplier and the retailer can benefit from using a designed contract. Our analysis suggests that the contract-implementing Pareto zone moves up from RR, R, to RD in terms of the revenue sharing rate. In other words, the supplier has more negotiation power in Scenario RD while the retailer has the advantage in Scenario RR. The profits of the supplier and the retailer in the coordinated scenarios depend on the revenue sharing rate. We also show that the supplier may encounter a hazard when adding a direct channel to Scenario R given that both Scenarios R and RD are coordinated.

This paper has its limitations. First, while this paper has focused on the aforementioned four channel structures, a further comparison with other channel structures with and without coordination could be informative. For example, extension from a single-supplier case as investigated in this paper to a two-supplier case is workable and might introduce additional insights about the channel-adding and contract-implementing Pareto zones. A theoretical difficulty might arise though, since defining the objective of coordination (e.g., optimizing the entire supply chain or some specific channels) could become challenging, especially when there exist multiple suppliers and common retailers. Moreover, according to Jeuland and Shugan (1983), a player has an incentive to defect from a contract at the expense of the cooperation partner(s). One can refer to Ingene and Parry (2004) for more open research venues on channel distribution and coordination.

This paper utilized the revenue sharing contract to demonstrate the impact of channel coordination in terms of contract-implementing Pareto zones. As a matter of fact, we believe that other contract formats, such as quantity discount schedule (Jeuland and Shugan, 1983), two-part tariffs (Ingene and Parry, 1995b,a), price-discount-sharing contract (Bernstein and Federgruen, 2005), and others, can also coordinate supply chains like those in our paper. Although each of them have different pros and cons, it would be interesting to explore the differences in coordination dynamics among these contract formats.

Due to the focus of this paper, we ignored some other significant marketing factors, such as service and quality guarantee, among others. Indeed, these factors can be incorporated into our base model, although these potential changes would shift the focus and likely require much more demanding computations. Additionally, suppliers and retailers are implementing different online strategies, such as free shipping, online rebates/coupons, and direct shipping; however, how these strategies interact with channel distribution decisions is unclear. Furthermore, it is widely acknowledged that online service quality is inferior to local retail stores, which might be an advantage for retailers to exploit; however, whether suppliers will overcome this barrier in their online direct channels remains unknown. Moreover, product differentiation, customization, and assortment in multi-channel environments need further investigation.

Last, but not least, this paper employs the elegant linear demand function with an underlying utility function proposed in Ingene and Parry (2004) (Chapter 11) and Ingene and Parry (2007). We echo their suggestions that an investigation of a more general demand function would be useful. Furthermore, how demand uncertainty affects the channel selection and coordination is another interesting research venue.

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**Appendix A.**

**Proof of Theorem 1.** We first summarize the computation results of Scenarios R, D, and RD in Table A.1. We define
Table A.1 Optimal solutions in Scenarios R, D, and RD without coordination.

<table>
<thead>
<tr>
<th>Scenario R</th>
<th>Scenario D</th>
<th>Scenario RD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w^* )</td>
<td>( \frac{1}{4} (\alpha_r - c_r) )</td>
<td>( \frac{8}{3} (\alpha_r - c_r) (\alpha_s - c_s) )</td>
</tr>
<tr>
<td>( p^*_r )</td>
<td>( -\frac{3}{8} \alpha_r + c_s )</td>
<td>( -\frac{4}{3} (\alpha_r - c_r) (\alpha_s - c_s) )</td>
</tr>
<tr>
<td>( p^*_s )</td>
<td>( \frac{1}{2} (\alpha_s + c_s) )</td>
<td>( \frac{2}{8} (\alpha_r - 2\alpha_s + \alpha_s) )</td>
</tr>
<tr>
<td>( \Pi^*_r )</td>
<td>( \frac{2}{8} (\alpha_r - c_r)^2 )</td>
<td>( \frac{2}{8} (\alpha_r - c_r)^2 )</td>
</tr>
<tr>
<td>( \Pi^*_s )</td>
<td>( \frac{2}{8} (\alpha_r - c_r)^2 )</td>
<td>( \frac{2}{8} (\alpha_r - c_r)^2 )</td>
</tr>
</tbody>
</table>

\( \Delta \Pi^*_1 | \text{RD} - R \equiv \Pi^*_1 | \text{RD} - \Pi^*_1 | R. \) Based on the results from Table A.1, we first obtain

\[ \Delta \Pi^*_1 | \text{RD} - R = (\alpha_s - c_s)^2 \times \text{Temp}, \]

where

\[ \text{Temp} = \frac{1}{4} + \frac{(\Omega - \theta)^2}{8 - 7\theta^2 - \theta^4} - \frac{1}{8} \Omega^2. \]

From the nonnegative demand constraints that

\[ D^*_r = \frac{(2 + \theta^2) (\alpha_r - c_r - \theta (\alpha_s - c_s))}{-7\theta^2 - \theta^4} \geq 0, \]

\[ D^*_s = \frac{(8 - \theta^2 - \theta^4) (\alpha_s - c_s) - 6\theta (\alpha_r - c_r)}{2 (8 - 7\theta^2 - \theta^4)} \geq 0, \]

\[ \Pi^*_{1-RR} = \frac{\left(2 - \theta^2\right) c_r - \theta c_s - (2 - \theta^2) \alpha_s + \theta \alpha_s}{4 (4 - \theta^2)^2 (1 - \theta^2)}, \]

\[ \Pi^*_{2-RR} = \frac{(\theta c_r - (2 - \theta^2) c_s + 2\alpha_s - \theta (\alpha_r + \theta \alpha_s))^2}{4 (4 - \theta^2)^2 (1 - \theta^2)}, \]

\[ \Pi^*_{3-RR} = \frac{2c_r (\theta c_s + (2 - 2\theta) \alpha_r - \theta \alpha_s) + 2c_s (\theta \alpha_r - (2 - \theta) \alpha_s)}{4 (4 - 5\theta^2 + \theta^4)}. \]

we must have \( \theta \leq \Omega \leq ((8 - \theta^2 - \theta^4)/6\theta). \) We can further show that Temp reaches its minimum (i.e., \( \text{Temp} = ((3 + \theta^2)/(28 + 4\theta^2)) > 0 \) at \( \Omega = 8/7(\theta + \theta^2) \) and is convex increasing in \( \Omega \) in the entire feasible domain as defined by the above nonnegative demand constraints. Thus, we prove that, for the supplier, Scenario RD always (weakly) outperforms Scenario R in the feasible domain.

For the retailer, we have

\[ \Pi^*_{1-RD} - \Pi^*_{1-R} = \frac{\left(2 + \theta^2\right) c_r - \theta c_s - \alpha_r + \theta \alpha_s}{4 (1 - \theta^2)(8 + \theta^2)^2} - \frac{1}{16} (\alpha_r - c_r)^2. \]

Reorganizing the equation, we can show that \( \Pi^*_{1-RD} - \Pi^*_{1-R} \geq 0 \) is equivalent to

\[ \frac{(8 + 4\theta^2)^2}{(1 - \theta^2)(8 + \theta^2)^2} \left(1 - \frac{\theta}{\Omega}\right) ^2 > 1. \]

After some algebra, we can obtain \( \hat{\Omega} \) and Eq. (7), where \( \hat{\Omega} \) is decreasing in \( \theta \), which can be illustrated graphically.

**Proof of Theorem 2.** Comparing the supplier’s profits of Scenarios RD and D in Table A.1 results in

\[ \Pi^*_{1-RD} - \Pi^*_{1-D} = \frac{\left(\theta (\alpha_s - c_s) - (\alpha_r - c_r)\right)^2}{8 - 7\theta^2 - \theta^4} \]

which is always nonnegative.

**Proof of Lemma 1.** Similar to that in Scenario RD, we work the process by solving the optimal retail prices in a Nash game from the first-order conditions of Eqs. (9) and (10), and then solving the wholesale prices simultaneously from the first-order conditions of Eq. (11), after replacing the prices with the optimal prices obtained from the former step. The optimal solution is given as follows:

\[ w^*_r = \frac{1}{2} (\alpha_r - c_r), \]

\[ w^*_s = \frac{1}{2} (\alpha_s - c_s), \]

\[ p^*_r = \frac{\theta c_r + 2c_s - \theta \alpha_r + 2(3 - \theta^2) \alpha_s}{2(4 - \theta^2)}, \]

\[ p^*_s = \frac{2c_r + \theta c_s + 2(3 - \theta^2) \alpha_r - \theta \alpha_s}{2(4 - \theta^2)}. \]

The corresponding profits for Retailer r, r2, and the supplier are

\[ \Pi^*_{1-RR} = \left(\alpha_r - c_r\right)^2 \times \text{Temp}, \]

As required, the optimal demand must be nonnegative.

\[ D^*_r = \frac{(2 - \theta^2) (\alpha_r - c_r - \theta (\alpha_s - c_s))}{2 (4 - 5\theta^2 + \theta^4)} \geq 0, \]

\[ D^*_s = \frac{(2 - \theta^2) (\alpha_s - c_s) - \theta (\alpha_r - c_r)}{2 (4 - 5\theta^2 + \theta^4)} \geq 0. \]

Thus, we must have \( \theta \leq \Omega \leq (1/\theta). \) It is obvious that \( D^*_r \) increases in \( c_r \) while \( D^*_s \) decreases in \( c_r \). We define \( \Delta \Pi^*_1 | RR - R \equiv \Pi^*_1 | RR - \Pi^*_1 | R \), which can be obtained by comparing the corresponding profits in Eq. (A.1) and Table A.1. We have

\[ \Delta \Pi^*_1 | RR - R = (\alpha_s - c_s)^2 \times \text{Temp} 1, \]
where

\[ Temp1 = \frac{(2 - \theta^2)(\Omega - 1)^2}{4(4 - 5\theta^2 + \theta^4)} + \frac{\Omega}{2(2 + \theta - \theta^2)} - \frac{1}{8} \Omega^2. \]

We can further show that \( Temp1 \) is convex and reaches its minimum (i.e., \( 1/(12 - 4\theta^2) > 0 \)) when \( \Omega = 2/(3\theta - \theta^2) \). Thus, we can conclude that for the supplier, Scenario RR always (weakly) dominates Scenario R in the feasible domain. For retailer \( r \), we have

\[ \Pi_{r-RR}^s - \Pi_{r-R}^s = (\alpha_s - c_s)^2 \times Temp2, \]

where

\[ Temp2 = \frac{(2 - \theta^2)\Omega - \theta^2}{4(4 - \theta^2)^2(1 - \theta^2)} - \frac{1}{16} \Omega^2. \]

We can show that \( Temp2 \) is convex and reaches its minimum (i.e., \(-1/(4(8 - 5\theta^2 + \theta^3)) < 0 \)) when \( \Omega = (8 - 4\theta^2)/(\theta - 5\theta^2 + \theta^3) \geq 1/(\theta) \). Given that \( Temp2 = -\theta^2(12 - 4\theta^2 + \theta^4)/(16\theta^2) \leq 0 \) at \( \Omega = \theta \), we can conclude that in the entire feasible domain, Scenario R always (weakly) dominates Scenario RR for the retailer. It is worth noting that this result is robust even if we relax the utility function of Eq. (1) to allow different rates of change of marginal utility between channels as follows:

\[ U = \sum_{i=s,r} \left( \alpha_i D_i - \frac{b_i D_i^2}{2} \right) - \theta D_i D_r - \sum_{i,s,r} P_i D_i. \]

Nevertheless, the assumption of the same \( b_i \) “seems to be an innocuous assumption” since “the product is the same at both retailers” (Ingene and Parry, 2004) (page 494). \( \square \)

**Proof of Theorem 3.** Based on the proofs of Theorem 1 and Lemma 1, we compare the profit of RR to that of D and obtain the following:

\[ \Pi_{s-RR}^s - \Pi_{s-D}^s = -\frac{(2 - 4\theta^2 + \theta^4) (\alpha_s - c_s)^2 - (2 - \theta^2) (\alpha_r - c_r)^2 + 2\theta (\alpha_r - c_r) (\alpha_s - c_s)}{4(4 - 5\theta^2 + \theta^4)}. \]

Since \( 4 - 5\theta^2 + \theta^4 \geq 0 \), \( \Pi_{s-RR}^s - \Pi_{s-D}^s \geq 0 \) as long as

\[ (2 - 4\theta^2 + \theta^4) (\alpha_s - c_s)^2 - (2 - \theta^2) (\alpha_r - c_r)^2 + 2\theta (\alpha_r - c_r) (\alpha_s - c_s) \leq 0. \]

Reorganizing the above equation, we can show that it is equivalent to \( tempF = (2 - \theta^2) \Omega^2 - 2\theta \Omega - (2 - 4\theta^2 + \theta^4) \geq 0 \).

Further, after solving \( tempF = 0 \), we can infer that the above condition is equivalent to

\[ \Omega \geq \frac{\theta + (1 - \theta^2) \sqrt{(4 - \theta^2)}}{2 - \theta^2} \quad \text{or} \quad \Omega \leq \frac{\theta - (1 - \theta^2) \sqrt{(4 - \theta^2)}}{2 - \theta^2}. \]

Since \( \Omega \leq (\theta - (1 - \theta^2) \sqrt{(4 - \theta^2)}/(2 - \theta^2) \) is infeasible as required by the nonnegative demand constraint, we can conclude that, for the supplier, RR outperforms D if

\[ \Omega \leq \frac{\theta + (1 - \theta^2) \sqrt{(4 - \theta^2)}}{2 - \theta^2}. \]

Otherwise (\( \Omega < \Omega_2 \)), D outperforms RR. Note that \( \Omega_2 > 1 \) for any \( \theta \in [0, 1] \). Graphically, we can show that \( \Omega_2 \) is first increasing before \( \theta = 0.5 \), and then decreasing after that. \( \square \)

**Proof of Theorem 4.** For the supplier, the observation of \( RR > R \) is directly from Lemma 1. The result of \( D > RR \) can be obtained from Theorem 3 since \( \Omega = 1 < \Omega_2 \). Theorem 2 supports \( RD > D \) for the supplier. For the retailer, we can derive \( R > RR \) from Lemma 1. That \( D \) performs the worst is derived from the fact that the retailer earns zero in Scenario D. Compare the profits of Scenarios RR and RD from the proofs of Theorem 1 and Lemma 1. Under the symmetric setting, we obtain the following results and prove the theorem.

\[ \Pi_{s-RR}^s - \Pi_{s-RD}^s = \frac{(1 - \theta)(64 - 64\theta + 64\theta^2 - 31\theta^3 + 16\theta^4 - 4\theta^5)(c_r - c_s)^2}{4(2 - \theta)(1 + \theta)(8 + \theta)^2} \geq 0, \]

\[ \Pi_{s-RR}^s - \Pi_{s-RD}^s = -\frac{8 - 4\theta - 4\theta^2 + \theta^3 - \theta^4}{4(8 + \theta^2)(2 + \theta - \theta^2)}(c_r - c_s)^2 \leq 0. \]

**Proof of Lemma 2.** In Scenario R, we apply \( w(\delta) = \delta - \rho c_r \), where \( \delta = 0 \), into its profit function, and then the retailer finds the optimal retail price to optimize its own profit, which simultaneously coordinates the supplier chain. The optimal retail price is given by

\[ P_r^* = \frac{1}{2}(\alpha_r + c_r). \]

The coordinated profits of the supplier, the retailer, and the entire supply chain in Scenario R are given by

\[ \Pi_{r-R}^s = \frac{(1 - \rho)(\alpha_r - c_r)^2}{4}, \]

\[ \Pi_{r-R}^s = \frac{\rho(\alpha_r - c_r)^2}{4}, \]

\[ \Pi_R^s = \frac{(\alpha_r - c_r)^2}{4}. \]

Comparing the above profits with the uncoordinated Scenario in Table A.1 results in Eq. (14) where both the supplier and the retailer are mutually benefitted by applying the coordination policy. \( \square \)

**Proof of Theorem 5.** We utilize a combination of the revenue sharing contact and a pricing scheme to coordinate Scenario RD.
The optimal channel prices are

$\delta^*_s$ and $\delta^*_r$.

1. Apply $w(\delta) = \delta - \rho c_i$ and $P_r = P_i + \delta$ to Eq. (5). Meanwhile, the retailer shares $\rho$ percentage of its revenue with the supplier;
2. Retailer $r$ finds $P^*_r$ to optimize its own profit;
3. Find $\delta^*$ and $\epsilon^*$ to optimize the entire supply chain profit $\Pi$.

We can obtain the unique $\delta^*_R$ and $\epsilon^*_R$, which coordinate Scenario RD, where

$$\delta^*_R = \frac{\theta(1 - \rho)(c_r - c_s - \alpha_r + \alpha_s)}{2(1 - \theta)}$$

and

$$\epsilon^*_R = \frac{1}{2}(c_r - c_s - \alpha_r + \alpha_s).$$

The optimal channel prices are

$P^*_s = \frac{1}{2}(\alpha_s + c_s), \quad P^*_r = \frac{1}{2}(\alpha_r + c_r)$.

The corresponding optimal profits of Scenario RD are

$$\Pi^*_R = \frac{(1 - \rho)(c_r - \delta c_s - \alpha_r + \delta \alpha_s)^2}{4(1 - \theta)^2(1 + \theta)},$$

$$\Pi^*_S = \frac{\Pi^*_R - \Pi^*_R}{4},$$

$$\Pi^*_R = \frac{c_r^2 + c_s^2 + \alpha_s^2 + \alpha_r^2 + 2c_r(\theta \alpha_r - \alpha_s) - 2\theta \alpha_r \alpha_s}{4 (1 - \theta^2)}.$$

These profits are exactly the same as those of integrated Scenario RD. Due to lengthy equations, we do not show the joint concavity of $\Pi_R$ with respect to $\delta$ and $\epsilon$. Comparing the above profits with the uncoordinated profits in Table A.1, we obtain a single boundary value of $\rho$ for each comparison such that the retailer can be better off with coordination if $\rho < \rho_{RD}$, and so can the supplier if $\rho > \rho_{RD}$, where

$$\rho_{RD} = \frac{4 + 4\theta + 6\theta^2}{8 + \theta^2} \quad \text{and} \quad \rho_{RD} = \frac{48 + 16\theta + 16\theta^2 - 3\theta^4 + 4\theta^5}{(8 + \theta^2)^2}.$$

Proof of Theorem 6. To coordinate Scenario RR, we first apply the revenue sharing contract and $w_i = \delta_i - \rho c_i$, $i = r, s$ into the profit functions of the retailers. We then solve $\{P^*_r, P^*_s\}$ for the retailers in a Nash game, and simultaneously find $\delta_r$ and $\delta_s$ to maximize the overall profit of the entire supply chain. Consequently, we obtain $\delta^*_r = (1/2)\theta(1 - \rho)(\alpha_r - c_r)$ and $\delta^*_s = (1/2)\theta(1 - \rho)(\alpha_s - c_s)$. The solution of the above $\delta^*_r$ and $\delta^*_s$ is unique. The corresponding optimal prices are given as follows:

$P^*_s = \frac{1}{2}(\alpha_s + c_s), \quad P^*_r = \frac{1}{2}(\alpha_r + c_r)$.

The profits of Retailer $r, r, 2$, the supplier, and the entire supply chain are given by

$\Pi^*_R = \frac{(1 - \rho)(c_r - \delta c_s - \alpha_r + \delta \alpha_s)^2}{4(1 - \theta^2)},$

$\Pi^*_S = \frac{(1 - \rho)(c_r - c_s - \alpha_r + \alpha_s)^2}{4(1 - \theta^2)},$

$\Pi^*_R = \frac{(c_r - \delta c_s - \alpha_r + \delta \alpha_s)(\theta c_r + (\theta - \delta) c_s - \rho c_s - (1 - \rho) c_s)}{4 (1 - \theta^2)},$

$\Pi^*_R = \frac{\delta^*_r = \frac{c_r^2 + 2c_r(\theta \alpha_r - \alpha_s) - 2c_s(\theta \alpha_r - \alpha_s)}{4 (1 - \theta^2)}}{4(1 - \theta^2)}.$

We then compare the supplier’s and two retailers’ profits as shown above with those in uncoordinated RR as shown in the proof of Lemma 1, and we obtain the following lower and upper bounds where both the supplier and the retailers are better off in coordinated Scenario RR than in uncoordinated RR as follows.

$\rho_{RR} = \frac{(2 + 3\theta^2 - \theta^4)(1 + \Omega^2) - 2\theta (5 - \theta^2) \Omega}{4(3\theta^2 - \theta^4)(1 + \Omega^2) - 4\theta (4 - \theta^2) \Omega},$

$\rho^*_{RR} = 1 - \frac{(2 - \theta^2)^2 (\Omega - \theta^2)}{4(1 - \theta^2)^2 (\Omega - 1)^2}.$

Note that the feasible domain for $\Omega$ as imposed by nonnegative demands is $\Omega \in [\theta, 1/\theta]$. Thus, $\rho^*_{RR}$ and $\rho^*_{RR}$ are symmetric and have the same lower and upper bounds. While $\rho^*_{RR}$ increases in $\Omega$, $\rho_{RR}$ decreases in $\Omega$. To ensure that both retailers would adopt the coordination contract, we must have

$\rho_{RR} = \min_r \rho^*_{RR} = 1 - \frac{(2 - \theta^2)^2 (\Omega - \theta^2)}{4(1 - \theta^2)^2 (\Omega - 1)^2}.$

From the properties of $\rho^*_{RR}$ and $\rho_{RR}$, we find that $\rho_{RR}$ reaches its maximum at $\Omega = 1$. If $\Omega = 1$, we yield

$\rho_{RR} = \frac{1 - \theta}{2 \theta}$ and $\rho_{RR} = \frac{3 - 4\theta + \theta^2}{4 - 4\theta + \theta^2}.$

Proof of Theorem 7. As we obtained previously, in Scenario R, $\rho_{RR} = (1/2)$ and $\rho_{RR} = (3/4)$. From the proof of Theorem 6, we know that the highest value of $\rho_{RR}$ is given by

$\rho_{RR} = \frac{3 - 4\theta + \theta^2}{4 - 4\theta + \theta^2},$

which is no more than $(3/4)$ for any $\theta$. Meanwhile, $\rho_{RR}$ reaches its maximum at its upper bound $\Omega = (1/\theta)$ and the value is given by

$\rho_{RR} = \frac{2 - \theta^2}{4 - \theta^2},$

which is no more than $(1/2)$ for any $\theta$. Thus, we have $\rho_R - \rho_{RR} \geq 0$ and $\rho_R - \rho_{RR} \geq 0$. From our previous discussion in
Theorem 5, we can derive the following result that
\[
\rho_{RD} - \rho_R = \frac{\theta(8 - \theta + 2\theta^2)}{2(8 + \theta^2)} \geq 0
\]
and
\[
\bar{\rho}_{RD} - \bar{\rho}_R = \frac{\theta(64 - 48\theta + 64\theta^2 - 15\theta^3 + 16\theta^4)}{4(8 + \theta^2)^2} \geq 0
\]
for any \( \theta \in [0, 1] \). Overall, we have \( \rho_{RR} \leq \rho_R \leq \rho_{RD} \) and \( \bar{\rho}_{RR} \leq \bar{\rho}_R \leq \bar{\rho}_{RD} \). □

**Proof of Theorem 8.** The profit difference between coordinated Scenario RD and coordinated Scenario R from the proofs of Lemma 2 and Theorem 5 is given by
\[
\Delta \Pi_R | RD - R = \frac{1}{4}(1 - \rho) \left( (c_r - \theta c_s - a_r + \theta a_s)^2 \right) \frac{(1 - \theta^2)(1 + \theta)}{(1 + \theta)} \geq 0,
\]
\[
\Delta \Pi_s | RD - R = \frac{1}{4}(\theta c_r - c_s - \theta a_r + a_s)^2 \leq 0.
\]
Since \( \Delta \Pi_R | RD - R = (\Delta \Pi_s | RD - R) + (\Delta \Pi_s | RD - R) \), we must have \( (d \Delta \Pi_s | RD - R) \) ≤ 0 in its entire feasible domain as well. Solving \( \Delta \Pi_R | RD - R = 0 \) and \( \Delta \Pi_s | RD - R = 0 \) gives us the single crossing points \( \Omega_3 \) and \( \Omega_4 \) for the retailer and the supplier, respectively. Based on the monotonicity, we obtain Theorem 8. Graphically, we can show that both \( \Omega_3 \) and \( \Omega_4 \) (for any given \( \rho \)) are convex decreasing and converging to 1 (but not equal to 1, since \( \theta \neq 1 \)) as \( \theta \) grows. This implies that the channel-adding Pareto zone is narrowing as \( \theta \) grows. □

**References**


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